Stochastic bounds on performance measures for threshold-based queueing system with hysteresis : Application to cloud computing

Farah AIT SALAHT

INSTITUT TELECOM/ Télécom SudParis
Lab. SAMOVAR, 9, rue Charles Fourier, 91011 Evry Cedex, France

Joint Work with H. CASTEL (SAMOVAR, Telecom SudParis)

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OUTLINE

1 MOTIVATION

2 MODELING CLOUD DATA CENTER

3 STOCHASTIC BOUNDING SYSTEMS

4 NUMERICAL RESULTS

5 CONCLUSION

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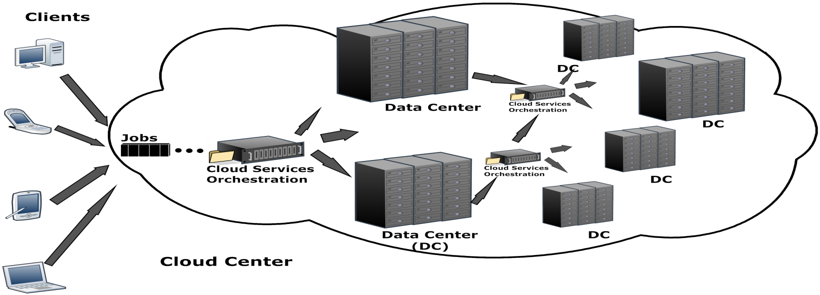
CLOUD COMPUTING - CLOUD ARCHITECTURES



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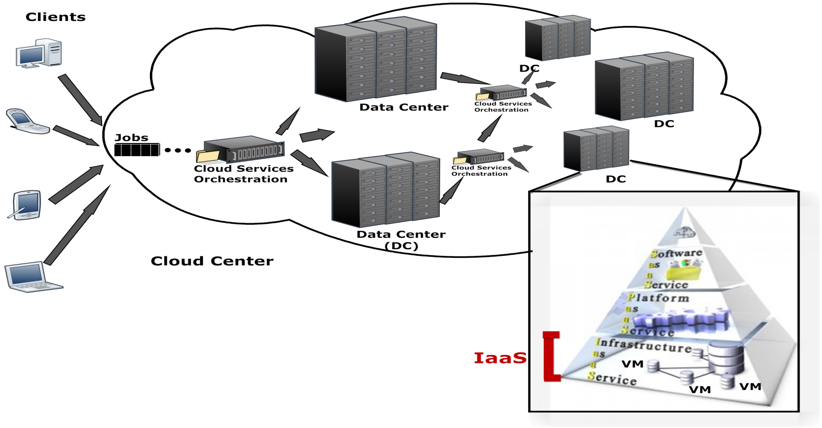

CLOUD COMPUTING - CLOUD ARCHITECTURES

BASIC IDEAS :

- A flexible architecture, a pool of virtualized resources depending on the demand
- Dynamicity of resources according to the scalability of the demand
- Load-Dependent Activation / Deactivation of VMs

GOAL :

- Definition of a queueing model which represents the dynamicity of the system
- Performance : Meeting SLA (Service Level Agreements) Requirements
- Reduce energy consumption (Energy Saving) and cost due to the activation of VMs

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MODELING CLOUD DATA CENTER

BASIC MODEL :

- Multi-Server Model for DC (Data Center)
- K homogeneous VMs
- Activation/Deactivation of VMs within each DC (state-dependent)
- Focus only on the number of jobs in DC

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- Multi-Server Model for DC (Data Center)
- K homogeneous VMs
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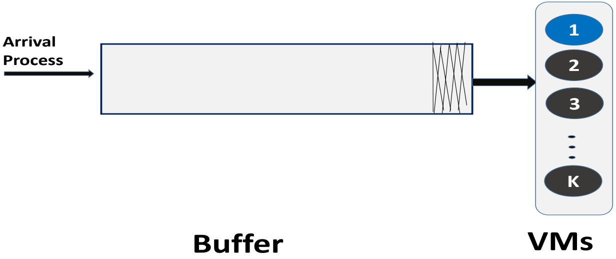
- Focus only on the number of jobs in DC
- **Threshold-based queueing system with hysteresis**
- Avoid costly and frequent oscillations around the threshold

MODELING CLOUD DATA CENTER

INDIVIDUAL DC MODEL

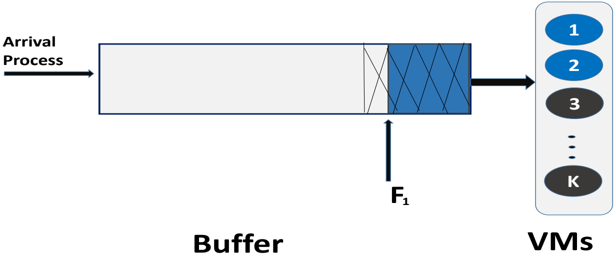
- K multi-server thresholds-based queueing system with hysteresis



MODELING CLOUD DATA CENTER

INDIVIDUAL DC MODEL

- K multi-server thresholds-based queueing system with hysteresis
- forward thresholds $(F_1, F_2, \ldots, F_{K-1})$ and reverse thresholds $(R_1, R_2, \ldots, R_{K-1})$



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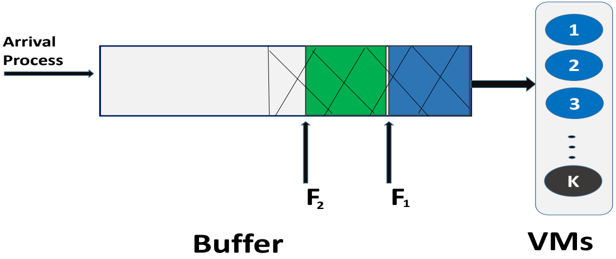

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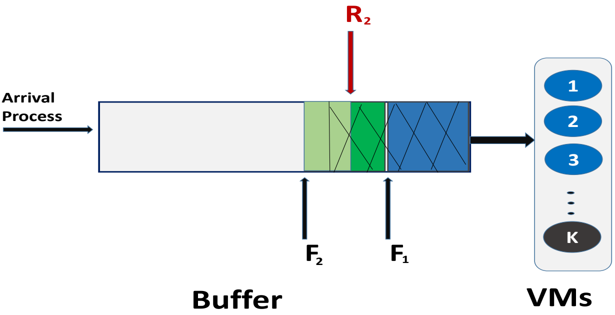
- $F_1 < F_2 < \ldots < F_{K-1}$, $R_1 < R_2 < \ldots < R_{K-1}$, and $R_i < F_i, \forall 1 \leq i \leq K-1$.



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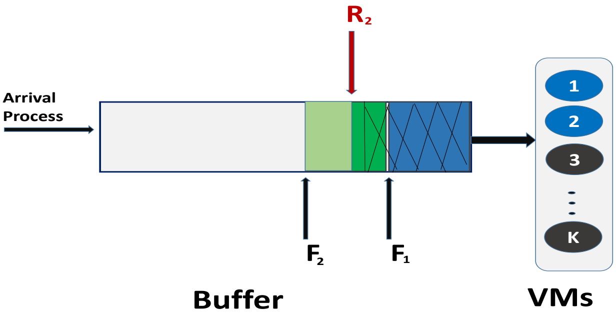

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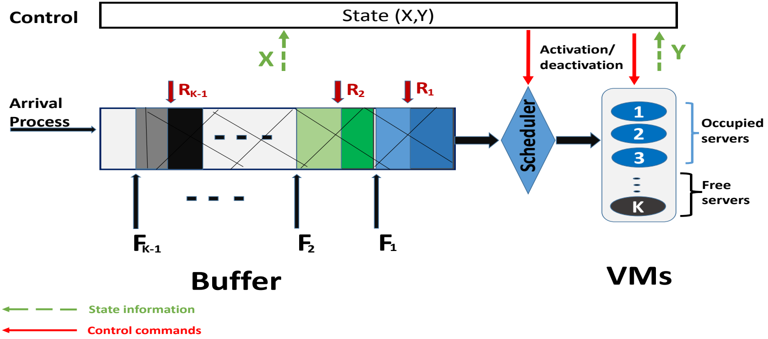

MODELING CLOUD DATA CENTER

Depending on the arrival process we distinguish the following two cases :

1. Homogeneous servers with Poisson arrivals
   - F. Aït-Salaht, H. Castel-Taleb. The threshold based queueing system with hysteresis for performance analysis of clouds. The 2015 IEEE International Conference on Computer, Information and Telecommunication Systems (CITS 2015), Gijón, Spain.
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MODELING CLOUD DATA CENTER

(2) Hysteresis queue with homogeneous servers and Batch-Arrival process

- Each arrival event corresponds to a batch arrival of size g_i , where $g_i = \Pr[\text{arrival of i jobs}], \quad i \geq 1.$

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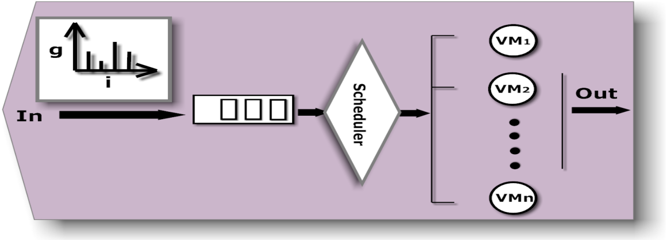

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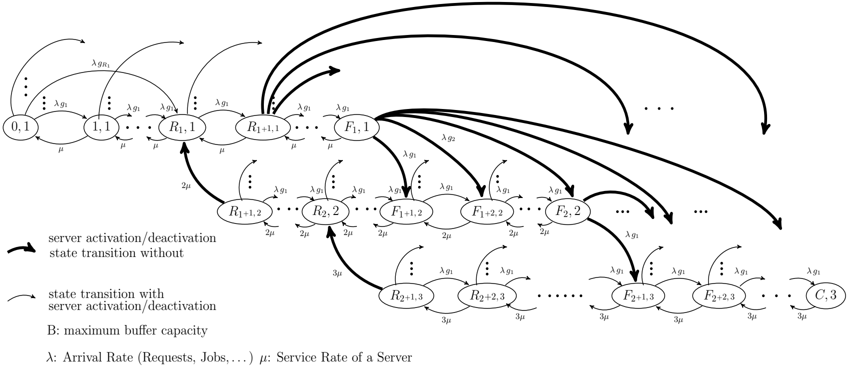

FIGURE : Example of state transition graph for a three-servers system

Modeling cloud data center

Threshold-based queueing system with hysteresis, homogeneous servers and Batch-Arrival process

- ***Numerical evaluation :***
  - Two-dimensional space of a continuous-time Markov chains
  - Mathematical analysis :
    - There may not exist a closed-form solution
    - Numerically too complex

Modeling cloud data center

Threshold-based queueing system with hysteresis, homogeneous servers and Batch-Arrival process

- ***Numerical evaluation :***
  - Two-dimensional space of a continuous-time Markov chains
  - Mathematical analysis :
    - There may not exist a closed-form solution
    - Numerically too complex
    - **Proposition : Use Stochastic bounds on Batch-arrival distribution**

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Stochastic bounding systems

Motivation :

- Reduce the complexity of the model
- Control of the complexity and Trade-off between accuracy and complexity (*by changing distribution sizes*)

Methodology :

- Use stochastic bound theory to reduce the size of the input distribution
- Stochastic bound : a bound of the exact distribution
- It implies **Bounds** on performance measures which are non decreasing rewards

A Brief Introduction to Stochastic Ordering

- $\mathcal{G} = 1, 2, \ldots, n$ a finite state space, X, Y : discrete distributions over $\mathcal{G}$, $p_X(i) = prob(X = i)$ and $p_Y(i) = prob(Y = i)$ for $i \in G$.
- **Definition of $\leq_{st}$ order :**

$$X \leq_{st} Y \text{ iff } \sum_{k=i}^{n} p_X(k) \leq \sum_{k=i}^{n} p_Y(k), \quad \forall i$$

- Comparison of non decreasing rewards :

$$X \leq_{st} Y \Longleftrightarrow E[f(X)] \leq E[f(Y)]$$

  for all non decreasing functions f, whenever expectations exist.
- Let F_X and F_Y be the cumulative distribution fonctions of X and Y respectively. Then,

$$X \leq_{st} Y \Longleftrightarrow F_X(a) \geq F_Y(a); \; \forall a \in \mathcal{G}$$

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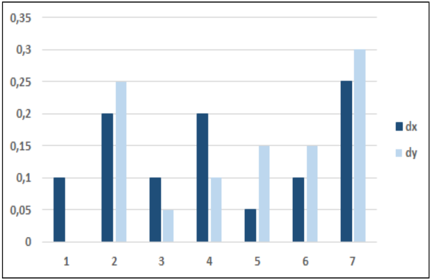
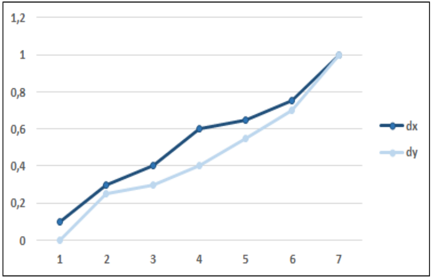


-The pmfs of a discrete distributions X and Y-



-Cumulative distribution functions-

FIGURE : $\mathcal{G} = \{1, 2, \ldots, 7\}$, $p_X = [0.1, 0.2, 0.1, 0.2, 0.05, 0.1, 0.25]$ et $p_Y = [0, 0.25, 0.05, 0.1, 0.15, 0.15, 0.3]$.

Stochastic bounds on distributions

- Hypothesis : totally ordered state space G, with length N
- We have Discrete distribution $\boldsymbol{d}$ and positive increasing reward $\boldsymbol{r}$, with $R[\boldsymbol{d}] = \sum \boldsymbol{r}(i)\boldsymbol{d}(i)$
- **Compute *d1* et *d2* such that :**
  1. $\boldsymbol{d}2 \leq_{st} \boldsymbol{d} \leq_{st} \boldsymbol{d}1$,
  2. $\boldsymbol{d}1$ and $\boldsymbol{d}2$ have only $K << N$ states (not necessarily the same set) ; ***d1*** has a support $\mathscr{H}^u$ and ***d2*** has a support $\mathscr{H}^l$,
  3. $\sum_{i \in \mathscr{H}} \boldsymbol{r}(i)\boldsymbol{d}(i) - \sum_{i \in \mathscr{H}^l} \boldsymbol{r}(i)\boldsymbol{d}2(i)$ is minimal among the lower bounding distributions of $\boldsymbol{d}$ with K states,
  4. $\sum_{i \in \mathscr{H}^u} \boldsymbol{r}(i)\boldsymbol{d}1(i) - \sum_{i \in \mathscr{H}} \boldsymbol{r}(i)\boldsymbol{d}(i)$ is minimal among the upper bounding distributions of $\boldsymbol{d}$ with K states,

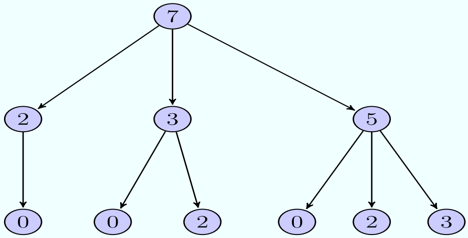
Optimal bounds , dynamic Programming

- Graph theory problem
- We consider a weighted graph $G = (V, E)$ with :
    - **Lower bound :** $w(e) = \sum_{j \in \mathscr{H}: u<j<v} \boldsymbol{d}(j)(\boldsymbol{r}(j) - \boldsymbol{r}(u))$
    - **Upper bound :** $w(e) = \sum_{j \in \mathscr{H}: u<j<v} \boldsymbol{d}(j)(\boldsymbol{r}(v) - \boldsymbol{r}(j))$
- Compute optimal bound $\equiv$ Compute a path of length K ($K << N$) with minimum cost in a graph G. Algorithm based on dynamic programming with complexity : $O(N^2K)$.
- The probability mass of deleted nodes is summed with
    - **Lower bound :** immediate predecessors
    - **Upper bound :** immediate successors

Example : Optimal upper bound

$\mathscr{A}$: is a discrete distribution defined on $\{0, 2, 3, 5, 7\}$
with probability vector $[0.05, 0.3, 0.15, 0.2, 0.3]$.
$\boldsymbol{r}$: reward function, $\boldsymbol{r}(a_i) = a_i$, $R[\mathscr{A}] = \sum_{a_i \in \mathbf{A}} \boldsymbol{r}(a_i)\, p_{\mathbf{A}}(i) = 4.15$.

- Compute the Optimal Upper Bound $\overline{\mathscr{A}}$ on 3 states such that $R[\overline{\mathscr{A}}] - R[\mathscr{A}]$ is minimal.



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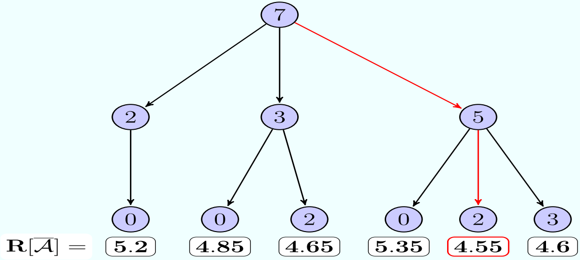

Bounding distribution $[0.35, 0.35, 0.3]$ with support $\{2, 5, 7\}$ and $R[\overline{\mathscr{A}}] = 4.55$.

THEORETICAL RESULTS

STOCHASTIC COMPARISON

We denote by :

- $X(t)$: Markov chain of Hysteresis model
- $X^u(t)$: Markov chain associated to the Hysteresis model with upper bound batch-arrival distribution
- $X^l(t)$: Markov chain associated to the Hysteresis model with lower bound batch-arrival distribution

The stochastic comparisons of processes by mapping functions is defined as follows :

1. $g(X(0)) \leq_{st} g(X^u(0)) \Rightarrow g(X(t)) \leq_{st} g(X^u(t)),\ t > 0.$
2. $g(X^l(0)) \leq_{st} g(X(0)) \Rightarrow g(X^l(t)) \leq_{st} g(X(t)),\ t > 0.$

- ***Stochastic bound on arrival process $\Longrightarrow$ bound on performance measures***

NUMERICAL EVALUATION

- We have to solve the steady-state distribution of the chains
  - Use solution techniques (GTH, Power Method...) on CTMC transition matrix
  - Use the fundamental solutions of Lui/Golubchik by Stochastic Complement Analysis
- Bounding systems are Less complex when the size of input distribution is small (sparse matrices)

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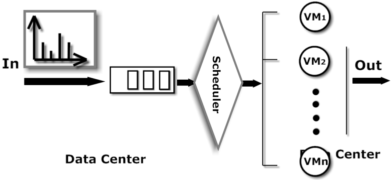
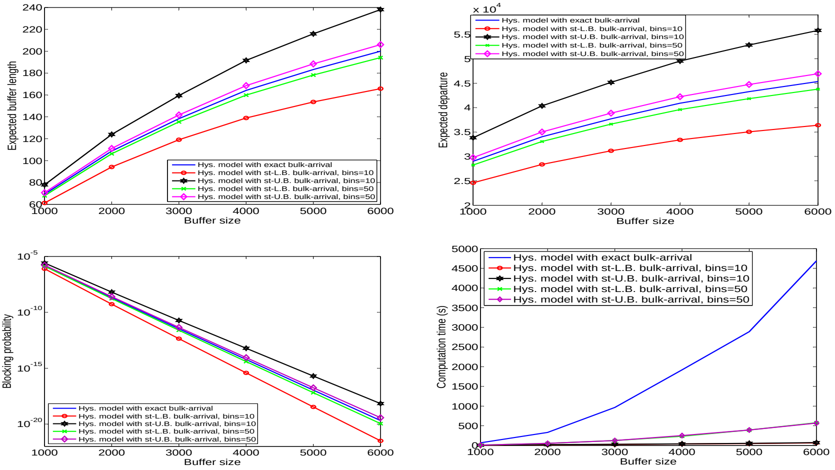
NUMERICAL RESULTS

- Threshold-based queueing system with hysteresis and batch-arrival, such that the distribution of the batch arrivals is randomly generated on a support $\{1, 2, 3, \ldots, 500\}$.



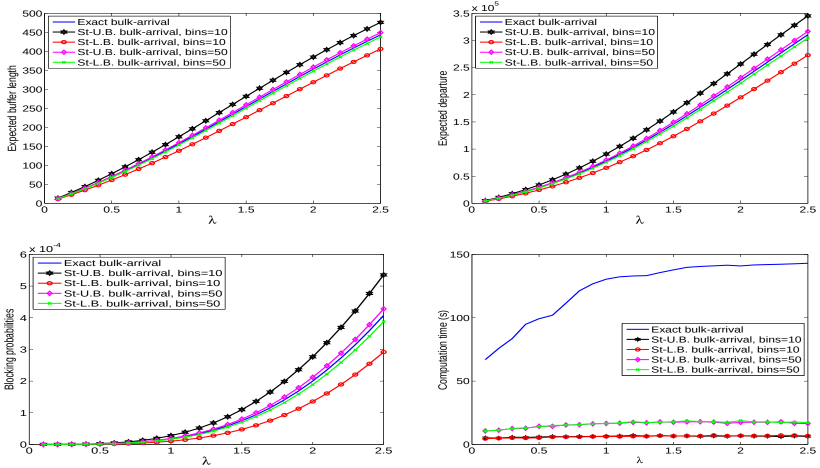
- Vary some input parameters :
  - Buffer size
  - Arrival rate
  - Degree of virtualization (number of servers)

Some performance measures versus buffer size



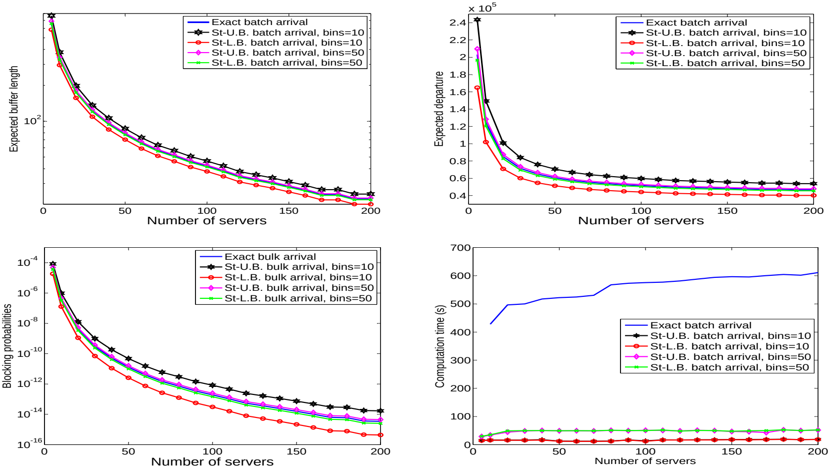
Number of servers is K = 10. For $B_1 = 1000$, $F_1 = [90, 140, 280, 400, 610, 690, 730, 840, 910]$ and $R_1 = [30, 90, 190, 270, 410, 510, 620, 700, 800]$. for $B = i \times 1000$, $F_i = i \times F$ and $R_i = i \times R$.

Some performance measures versus arrival rate



Number of servers is K = 10. For $B = 1000$, $F = (90, 140, 280, 400, 610, 690, 730, 840, 910)$ and $R = (30, 90, 190, 270, 410, 510, 620, 700, 800)$.

Some performance measures versus number of servers



$B = 2000$, $F = (\lfloor \frac{B}{K} \rfloor, 2 \times \lfloor \frac{B}{K} \rfloor, \ldots, (K-1) \times \lfloor \frac{B}{K} \rfloor)$ and $R_i = F_i - \lfloor \frac{B}{2K} \rfloor$, for $i = 1, \ldots, K-1$.

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CONCLUSION

- Represent the dynamicity of the resource according to the queue occupation (Hysteresis model)
- Propose to use stochastic ordering in order to derive bounds for the performance measures
- Illustrate the relevance of using stochastic bounds on batch-arrival distribution on performance measures and computation complexity
- Proposed stochastic bounding models are relevant for network dimensioning

Future work

- Define optimal threshold vectors
- Markov Modulated Arrivals
- Heterogeneous servers